Nonhomogeneous linear partial differential equations is the subject of Chapter 10. Chapter 11 is devoted to nonhomogeneous partial differential equations of mathematical physics. In particular, the methods of Chapter 10 are used to solve the equations:

(10) $\Delta V = f(x, y, z)$,

(11) $\Delta V - c^{-2}\, \partial^2 V/\partial t^2 = f(x, y, z, t)$,

(12) $\Delta V - k^{-1}\, \partial V/\partial t = f(x, y, z, t)$,

where Δ is the Laplacian, that is

(13) $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$.

The Laplacian is expressed in terms of some commonly used systems of orthogonal curvilinear coordinates, and if V is separable, then for each coordinate system the three resulting differential equations are presented. If $f(x, y, z)$ is separable and has a certain form in a given coordinate system, then a solution of (10) is also separable and the three resulting ordinary nonhomogeneous differential equations are set forth. Special cases relating this material to that of the earlier chapters are noted. Similar material for (11), (12) is also developed.

Y. L. L.

1. A. Erdélyi, W. Magnus, F. Oberhettinger & F. G. Tricomi, *Higher Transcendental Functions*, Vols. 1, 2, McGraw-Hill, New York, 1953.

2. G. N. Watson, *A Treatise on the Theory of Bessel Functions*, Cambridge Univ. Press, Cambridge, 1945.

3. Y. L. Luke, *Integrals of Bessel Functions*, McGraw-Hill, New York, 1962.

12 [7, 9].—M. Lal, *First* 39000 *Decimal Digits of* $\sqrt{2}$, deposited in UMT file.

Previous results on the decimal expansion of $\sqrt{2}$ to 19600D [1] are here extended to 39000D.

In the present investigation, the Newton-Raphson method was used to find an improved approximation to $\sqrt{2}$. Let x_1 be an approximate value of $\sqrt{2}$, then a value of x_2, accurate to twice the number of digits as x_1, is given by

$$x_2 = \tfrac{1}{2}\, x_1 + x_1^{-1} .$$

Here x_1 was known to 19600D, and in order to double the accuracy the reciprocal of x_1 must be carried to 2×19600 digits. The process of dividing 1 by x_1 was carried out on the 1620, Model II, 60K at Queen's University in two parts, and 39075D of x_2 were obtained. The accuracy of the value of x_2 was checked by squaring the output of 39076 digits with the decimal point disregarded. This multiplication, which was carried out in five sections of approximately 8K digits each, showed a 1 followed by an unbroken string of 39074 nines. This test establishes that this value of $\sqrt{2}$ is accurate to 39074D. The first 39000D are recorded here.

In order to examine the internal randomness of digits in an unsophisticated way, the frequency distribution of digits in 39 blocks of 1000 digits, and also in the total 39000 digits, was computed. The chi-square test for the goodness of fit reveals no abnormal behavior in the distribution of digits in this sample. These data are also recorded here.

Author's Summary

1. M. Lal, *Expansion of* $\sqrt{2}$ *to* 19600 *Decimals*, reviewed in *Math. Comp.*, v. 21, 1967, pp. 258–259, RMT 17.